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Axisymmetric vibrations of an orthotropic non-uniform cylindrical shell☆

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ABSTRACT

The axisymmetric vibrations of a circular cylindrical shell of constant thickness, orthotropic along the principal geometrical directions and non-uniform along the generatrices, are considered. It is shown, using a particular problem as an example, that the non-uniform physical-mechanical characteristics of the shell material may lead to qualitatively new results in problems of shell dynamics.

1. Formulation of the problem

Consider an orthotropic circular cylindrical shell of radius R and thickness h in a cylindrical system of coordinates (s, ϑ, γ). The physical-mechanical characteristics of the shell material, namely, the moduli of elasticity $E_i(s)$, Poisson's ratios $\nu_i(s)$, and the transverse shear modulus $G_{s\gamma} = G(s)$, are specified functions of the longitudinal coordinate. The problem is axisymmetric and all the non-zero required quantities depend solely on the coordinates s and γ and the time t.

We take as a basis the hypotheses of the improved theory of shells,[1,2] according to which it is assumed that

$$e_\gamma = 0, \quad \sigma_\gamma \approx 0, \quad \tau_{s\gamma} = \frac{1}{2}\left(\frac{h^2}{4} - \gamma^2\right)\varphi \tag{1.1}$$

where $\varphi = \varphi(s, t)$ is the required function, characterising the transverse shear.

Proceeding in the usual way, we obtain for the displacements[2]

$$u_s = u - \gamma\frac{\partial w}{\partial s} + \gamma\left(\frac{h^2}{8} - \frac{\gamma^2}{6}\right)a_{55}\varphi, \quad u_\vartheta = 0, \quad u_\gamma = w$$
$$a_{55} = a_{55}(s) = G^{-1} \tag{1.2}$$

where $u = u(s, t)$ and $w = w(s, t)$ are the required displacements. For the stresses, in addition to relations (1.1) we have

$$\sigma_s = B_{11}e_s + B_{12}e_\vartheta, \quad \sigma_\vartheta = B_{22}e_\vartheta + B_{12}e_s \tag{1.3}$$

where

$$B_{ii} = E_i(s)/\Delta, \quad B_{12} = \nu_1(s)E_2(s)/\Delta = \nu_2(s)E_1(s)/\Delta,$$
$$\Delta = 1 - \nu_1(s)\nu_2(s)$$
$$e_s = \frac{\partial u}{\partial s} - \gamma\frac{\partial^2 w}{\partial s^2} + \gamma\left(\frac{h^2}{4} - \frac{\gamma^2}{3}\right)\left(a_{55}\frac{\partial\varphi}{\partial s} + \frac{\partial a_{55}}{\partial s}\varphi\right), \quad e_\varphi = \frac{w}{R} - z\frac{w}{R^2}$$

Substituting the expressions for the internal forces and moment[1]

$$T_1 = B_{11}h\frac{\partial u}{\partial s} + B_{12}h\frac{w}{R}, \quad T_2 = B_{22}h\frac{w}{R} + B_{12}h\frac{\partial u}{\partial s}$$
$$M_1 = -B_{11}\frac{h^3}{12}\frac{\partial^2 w}{\partial s^2} - B_{12}\frac{h^3}{12}\frac{w}{R^2} + B_{11}a_{55}\frac{h^5}{120}\frac{\partial\varphi}{\partial s} + B_{11}\frac{\partial a_{55}}{\partial s}\frac{h^5}{120}\varphi,$$
$$N = \frac{h^3}{12}\varphi$$

into the equations of motion[2]

$$\frac{\partial T_1}{\partial s} = \rho h\frac{\partial^2 u}{\partial t^2} - \frac{\rho h^3}{12R}\frac{\partial^3 w}{\partial s\partial t^2} + \frac{a_{55}\rho h^5}{120R}\frac{\partial^2\varphi}{\partial t^2}, \quad T_2 - R\frac{dN}{\partial s} - \rho hR\frac{\partial^2 w}{\partial t^2}$$
$$\frac{\partial M_1}{\partial s} - N = \frac{\rho h^3}{12R}\frac{\partial^2 u}{\partial t^2} - \frac{\rho h^3}{12}\frac{\partial^3 w}{\partial s\partial t^2} + \frac{a_{55}\rho h^5}{120}\frac{\partial^2\varphi}{\partial t^2}$$

where $\rho = \rho(s)$ is the density of the shell material, we obtain a system of resolving differential equations in the three required functions

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$u(s,t)$, $w(s,t)$ and $\varphi(s,t)$

$$B_{11}\frac{\partial^2 u}{\partial s^2}+B_{12}\frac{1}{R}\frac{\partial w}{\partial s}+\frac{\partial B_{11}}{\partial s}\frac{\partial u}{\partial s}+\frac{\partial B_{12}}{\partial s}\frac{w}{R} = \rho\frac{\partial^2 u}{\partial t^2}-\frac{\rho h^2}{12R}\frac{\partial^3 w}{\partial s\partial t^2}+\frac{a_{55}\rho h^4}{120R}\frac{\partial^2\varphi}{\partial t^2}$$

$$B_{22}\frac{w}{R}+B_{12}\frac{\partial u}{\partial s}-R\frac{h^2}{12}\frac{\partial\varphi}{\partial s} = -\rho R\frac{\partial^2 w}{\partial t^2}$$

$$B_{11}\frac{\partial^3 w}{\partial s^3}+\frac{\partial B_{11}}{\partial s}\frac{\partial^2 w}{\partial s^2}+B_{12}\frac{1}{R^2}\frac{\partial w}{\partial s}+\frac{\partial B_{12}}{\partial s}\frac{w}{R^2}-$$

$$-B_{11}a_{55}\frac{h^2}{10}\frac{\partial^2\varphi}{\partial s^2}-\frac{\partial(B_{11}a_{55})}{\partial s}\frac{h^2}{10}\frac{\partial\varphi}{\partial s}-B_{11}\frac{\partial a_{55}}{\partial s}\frac{h^2}{10}\frac{\partial\varphi}{\partial s}-\frac{\partial}{\partial s}\left(B_{11}\frac{\partial a_{55}}{\partial s}\right)\frac{h^2}{10}\varphi =$$

$$= -\frac{\rho}{R}\frac{\partial^2 u}{\partial t^2}+\rho\frac{\partial^3 w}{\partial s\partial t^2}-\rho\frac{a_{55}h^2}{10}\frac{\partial^2\varphi}{\partial t^2} \tag{1.4}$$

Adding the boundary and initial conditions to this system, we obtain an initial-boundary-value problem, which describes the vibrations and wave processes in circular, non-uniform orthotropic cylindrical shells for axisymmetric strains.

System (1.4) simplifies considerably in the case of a weak orthotropic material, i.e. when the physical-mechanical characteristics of the material, on differentiation, behave as constants and we can neglect phenomena related to the transverse shear. Assuming that

$$\frac{\partial B_{11}}{\partial s}\approx 0,\quad \frac{\partial(B_{11}a_{55})}{\partial s}\approx 0,\quad \frac{\partial B_{12}}{\partial s}\approx 0,\quad a_{55} = 0$$

and then eliminating $\varphi(s,t)$ from the system, we obtain

$$B_{11}\frac{\partial^3 u}{\partial s^3}+\frac{B_{12}}{R}\frac{\partial^2 w}{\partial s^2} = \rho\frac{\partial^3 u}{\partial s\partial t^2}-\frac{h^2}{12R}\frac{\partial^4 w}{\partial s^2\partial t^2}$$

$$B_{11}\frac{\partial^4 w}{\partial s^4}+\frac{B_{12}}{R^2}\frac{\partial^2 w}{\partial s^2}+\frac{12}{h^2R^2}B_{22}w+\frac{12}{h^2R}B_{12}\frac{\partial u}{\partial s} =$$

$$= -\rho\frac{12}{h^2}\frac{\partial^2 w}{\partial t^2}+\rho\frac{\partial^4 w}{\partial s^2\partial t^2}-\frac{\rho}{R^2}\frac{\partial^3 u}{\partial s\partial t^2} \tag{1.5}$$

2. The vibrations of an isotropic circular cylindrical shell

Suppose

$$E_1 = E_2 = E_0 f(s),\quad \rho = \rho_0\psi(s),\quad \nu_1 = \nu_2 = \nu = \text{const} \tag{2.1}$$

We will mainly consider longitudinal steady harmonic vibrations,[3] ignoring transverse shears.

Suppose

$$u = u_0(s)e^{i\omega t},\quad w = w_0(s)e^{i\omega t}$$

Then, after reduction, from system (1.5) we obtain the following system of equations in u_0 and w_0

$$\frac{d}{ds}\left[\left(\frac{du_0}{ds}+\nu\frac{w_0}{R}\right)f\right] = -\frac{\rho_0 h}{C}\psi\omega^2 u_0,\quad C = \frac{E_0 h}{1-\nu^2}$$

$$\frac{d^2}{ds^2}\left(f\frac{d^2 w_0}{ds^2}\right)+\frac{12}{Rh^2}\left(\frac{w_0}{R}+\nu\frac{du_0}{ds}\right)f = 0 \tag{2.2}$$

We will consider the case of periodic non-uniformity of the shell material in the form[4]

$$f = 1+\varepsilon_1\sin\lambda_1 s,\quad \psi = 1+\varepsilon_2\sin\lambda_1 s;\quad \lambda_1 = \pi/l,\quad |\varepsilon_1|<1,\quad |\varepsilon_2|<1 \tag{2.3}$$

Then the solution of system (2.2) can be represented in the form of Fourier series

$$u_0 = a_0+\sum_{n=1}^{\infty}(a_n\cos\lambda_n s+b_n\sin\lambda_n s),$$

$$w_0 = c_0+\sum_{n=1}^{\infty}(c_n\cos\lambda_n s+d_n\sin\lambda_n s);\ \lambda_n = \frac{n\pi}{l} \tag{2.4}$$

Substituting these series into relations (2.2) and equating the sums of the free terms and also the coefficients of $\cos\lambda_n s$ and $\sin\lambda_n s$ to zero, we obtain the free terms

$$a_0 = -\varepsilon_2 b_1/2,\quad c_0 = -\varepsilon_1(d_1-\nu R\lambda_1 a_1)/2 \tag{2.5}$$

and also, taking expression (2.5) into account, an infinite system of sequential homogeneous equations for determining the remaining coefficients of series (2.4):

for $n=1$

$$\left(\eta_1-1-\frac{\varepsilon_1\nu^2}{2}\right)a_1+\frac{\nu}{\varsigma_1}\left(1-\frac{\varepsilon_1^2}{2}\right)d_1+\frac{\varepsilon_2}{2}\eta_1 b_2+\frac{\varepsilon_1\nu}{2\varsigma_1}c_2 = 0$$

$$-\nu\varsigma_1\left(1-\frac{\varepsilon_1}{2}\right)a_1+\left(1-\frac{\varepsilon_1^2}{2}+\xi_1\varsigma_1^2\right)d_1-\varepsilon_2\nu\varsigma_1 b_2-\frac{\varepsilon_1}{2}c_2 = 0$$

$$\left[\left(1-\frac{\varepsilon_2}{2}\right)\eta_1-1\right]b_1-\frac{\nu}{\varsigma_1}c_1-\frac{\varepsilon_2}{2}\eta_1 a_2-\frac{\varepsilon_1\nu}{2\varsigma_1}c_2 = 0$$

$$\nu\varsigma_1 b_1+(1+\xi_1\varsigma_1^2)c_1-\varepsilon_1\nu\varsigma_1 a_2+\frac{\varepsilon_1}{2}d_2 = 0 \tag{2.6}$$

for $n\geq 2$ (two unwritten equations are obtained with the replacement of the symbols indicated)

$$(\eta_n-1)a_n+\frac{\nu}{\varsigma_n}d_n-\frac{\varepsilon_2}{2}\eta_n(b_{n-1}-b_{n+1})-\frac{\varepsilon_1\nu}{2\varsigma_n}(c_{n-1}-c_{n+1}) = 0$$

$$-\nu\varsigma_n a_n+(1+\xi_n\varsigma_n^2)d_n+\frac{\varepsilon_1}{2}\nu\varsigma_n\left[\left(1-\frac{1}{n}\right)b_{n-1}-\left(1+\frac{1}{n}\right)b_{n+1}\right]+$$

$$+\frac{\varepsilon_1}{2}\left[1+\left(1-\frac{1}{n}\right)^2\xi_n\varsigma_n^2\right]c_{n-1}-\frac{\varepsilon_1}{2}\left[1+\left(1+\frac{1}{n}\right)^2\xi_n\varsigma_n^2\right]c_{n+1} = 0$$

$$(a,b,c,d)\to(-b,a,-d,-c) \tag{2.7}$$

The following notation has been used

$$\eta_n = \frac{\rho_0 h\omega^2}{c\lambda_n^2},\quad \xi_n = \frac{h^2\lambda^2}{12},\quad \varsigma_n = R\lambda_n \tag{2.8}$$

where η_n are dimensionless characteristics of the square of the required phase velocities and ω/λ_n is the phase velocity.

In the first approximation, the truncated system of four equations in the constants a_1, b_1, c_1 and d_1 is decomposed into two independent systems of two equations

$$\left(\eta_1-1-\frac{\varepsilon_1\nu^2}{2}\right)a_1+\frac{\nu}{\varsigma_1}\left(1-\frac{\varepsilon_1^2}{2}\right)d_1 = 0,$$

$$-\nu\varsigma_1\left(1-\frac{\varepsilon_1}{2}\right)a_1+\left(1-\frac{\varepsilon_1^2}{2}+\xi_1\varsigma_1^2\right)d_1 = 0 \tag{2.9}$$

$$\left[\left(1-\frac{\varepsilon_2}{2}\right)\eta_1-1\right]b_1-\frac{\nu}{\varsigma_1}c_1 = 0,\quad \nu\varsigma_1 b_1+(1+\xi_1\varsigma_1^2)c_1 = 0 \tag{2.10}$$

By equating the determinants of system (2.9) and (2.10) to zero we obtain the first two different vibration frequencies.

In the special case of a momentless shell ($\xi_1\varsigma_1^2 \ll 1$) we obtain, from the condition for the determinant of system (2.9) to be equal to zero,

$$\eta_{11} = 1-\nu^2(1-\varepsilon_1) \tag{2.11}$$

The corresponding value of the dimensionless frequency from system (2.10) has the form

$$\eta_{12} = (1-\nu^2)(1-\varepsilon_2/2)^{-1} \tag{2.12}$$

When $\varepsilon_1 = 0$ and $\varepsilon_2 = 0$ these frequencies, naturally, are identical with the first frequency of natural vibrations of momentless cylindrical shell.

In the second approximation, the system of equations in the constants a_1, d_1, b_2 and c_2 are also separated from the system of equations in b_1, c_1, a_2 and d_2, but now both systems contain the coefficients ε_1 and ε_2, representing the non-uniformity of the material both with respect to the modulus of elasticity and with respect to the density. In this case we have

$$\begin{gathered}\left(\eta_1 - 1 - \frac{\varepsilon_1\nu^2}{2}\right)a_1 + \frac{\nu}{\varsigma_1}\left(1-\frac{\varepsilon_1^2}{2}\right)d_1 + \frac{\varepsilon_2}{2}\eta_1 b_2 + \frac{\varepsilon_1\nu}{2\varsigma_1}c_2 = 0\\ -\nu\varsigma_1\left(1-\frac{\varepsilon_1}{2}\right)a_1 + \left(1-\frac{\varepsilon_1^2}{2}+\xi_1\varsigma_1^2\right)d_1 - \varepsilon_1\nu\varsigma_1 b_2 - \frac{\varepsilon_1}{2}c_2 = 0\\ \frac{\varepsilon_2}{2}\eta_2 a_1 + \frac{\varepsilon_1\nu}{2\varsigma_2}d_1 + (\eta_2-1)b_2 - \frac{\nu}{\varsigma_2}c_2 = 0\\ \frac{\varepsilon_1\nu}{4}\varsigma_1 a_1 - \frac{\varepsilon_1}{2}\left(1+\frac{\xi_2\varsigma_2^2}{4}\right)d_1 + \nu\varsigma_2 b_2 + (1+\xi_2\varsigma_2^2)c_2 = 0\end{gathered} \tag{2.13}$$

$$\begin{gathered}\left[\left(1-\frac{\varepsilon_2}{2}\right)\eta_1 - 1\right]b_1 - \frac{\nu}{\varsigma_1}c_1 - \frac{\varepsilon_2}{2}\eta_1 a_2 + \frac{\varepsilon_1\nu}{2\varsigma_1}d_2 = 0\\ \nu\varsigma_1 b_1 + (1+\xi_1\varsigma_1^2)c_1 - \varepsilon_1\nu\varsigma_1 a_2 + \frac{\varepsilon_1}{2}d_2 = 0\\ -\frac{\varepsilon_2}{2}\eta_2 b_1 - \frac{\varepsilon_1\nu}{2\varsigma_2}c_1 + (\eta_2-1)a_2 + \frac{\nu}{\varsigma_2}d_2 = 0\end{gathered} \tag{2.14}$$

$$\frac{\varepsilon_1\nu}{4}\varsigma_2 b_1 + \frac{\varepsilon_1}{2}\left(1+\frac{\xi_2\varsigma_2^2}{4}\right)c_1 - \nu\varsigma_2 a_2 + (1+\xi_2\varsigma_2^2)d_2 = 0$$

In the second approximation, the system of Eq. (2.13) in the constants a_1, d_1, b_2 and c_2 can also be separated from the system of Eq. (2.14) in the constants b_2, c_1, a_2 and d_2. Equating the determinants of these systems of equations to zero, we determine the vibration frequencies in the second approximation.

In the special case when $\varepsilon_1 = 0$, for a momentless shell ($\xi_1\varsigma_1^2 \ll 1$, $\xi_2\varsigma_2^2 \ll 1$) we obtain, for the condition from the determinant of system (2.13) to be equal to zero,

$$\eta_{11} \approx (1-\nu^2)(1-5\varepsilon_2^2/36), \quad \eta_{21} \approx 4(1-\nu^2)(1+\varepsilon_2^2/3) \tag{2.15}$$

while the values of the dimensionless frequency characteristic for system (2.14) have the form

$$\eta_{21} \approx (1-\nu^2)(1-\varepsilon_2/2), \quad \eta_{22} \approx 4(1-\nu^2)(1+\varepsilon_2/2) \tag{2.16}$$

For a momentless shell and with $\varepsilon_2 = 0$, the systems of Eqs. (2.13) and (2.14) reduce to the form

$$\begin{gathered}\left(\eta_1 - 1 - \frac{\nu^2\varepsilon_1}{\underline{2}}\right)a_1 + \frac{\nu}{\varsigma_1}\left(1-\frac{\varepsilon_1^2}{\underline{2}}\right)d_1 + \frac{\varepsilon_1\nu}{2\varsigma_1}c_2 = 0\\ -\nu\varsigma_1\left(1-\frac{\varepsilon_1}{\underline{2}}\right)a_1 + \left(1-\frac{\varepsilon_1^2}{\underline{2}}\right)d_1 - \varepsilon_1\nu\varsigma_1 b_2 - \frac{\varepsilon_1}{2}c_2 = 0\\ \frac{\varepsilon_1\nu}{2\varsigma_2}d_1 + (\eta_2-1)b_2 - \frac{\nu}{\varsigma_2}c_2 = 0, \quad \frac{\varepsilon_1\nu}{4}\varsigma_2 a_1 - \frac{\varepsilon_1}{2}d_1 + \nu\varsigma_2 b_2 + c_2 = 0\end{gathered} \tag{2.17}$$

$$\begin{gathered}(\eta_1-1)b_1 - \frac{\nu}{\varsigma_1}c_1 + \frac{\varepsilon_1\nu}{2\varsigma_1}d_2 = 0, \quad \nu\varsigma_1 b_1 + c_1 - \varepsilon_1\nu\varsigma_1 a_2 + \frac{\varepsilon_1}{2}d_2 = 0\\ -\frac{\varepsilon_1\nu}{2\varsigma_2}c_1 + (\eta_2-1)a_2 + \frac{\nu}{\varsigma_2}d_2 = 0, \quad -\frac{\varepsilon_1\nu}{4}\varsigma_2 b_1 + \frac{\varepsilon_1}{2}c_1 - \nu\varsigma_2 a_2 + d_2 = 0\end{gathered} \tag{2.18}$$

If we neglect the underlined terms in system (2.17), the determinants of systems (2.17) and (2.18) are identical.

If we equate the determinant of system (2.18) to zero we obtain the following equation for the required vibration frequency:

$$\eta_1^2 - 5(1-\alpha_1\nu^2)\eta_1 + 4[1-2\alpha_2\nu^2 + \alpha_2(1+\varepsilon_1^2\varsigma_1/4)\nu^4] = 0 \tag{2.19}$$

where

$$\alpha_1 = \left(1-\frac{\varepsilon_1^2}{20}\right)\left(1-\frac{\varepsilon_1^2}{4}\right)^{-1}, \quad \alpha_2 = \left(1+\frac{\varepsilon_1^2}{4}\right)\left(1-\frac{\varepsilon_1^2}{4}\right)^{-1}$$

When $\nu = 0.3$, $\varsigma_1 = 4$ in the dependence on the coefficient ε_1, characterizing the degree of non-uniformity, the numerical values of the two dimensionless vibration frequencies are as follows: $\eta_{11} = 0.91$ when $0 \le \varepsilon_1 \le 0.2$, $\eta_{11} = 0.90$, 0.89, 0.89 and 0.86 when $\varepsilon_1 = 0.3$, 0.4, 0.5, 0.6 and 0.7 respectively, and $\eta_{21} = 3.64$ when $0 \le \varepsilon_1 \le 0.7$.

In particular, when $\varepsilon_1 = 0$ and $\varepsilon_2 = 0$ we have $\eta_{11} = 1-\nu^2$, $\eta_{21} = 4(1-\nu^2)$.

3. Transverse harmonic vibrations ignoring transverse shears and rotational inertia

In the case considered, after conversions, we obtain the following system of differential equations in w_0 from system (1.5)

$$\frac{h^2}{12}\frac{d^2}{ds^2}\left(f\frac{d^2 w_0}{ds^2}\right) + \frac{1-\upsilon^2}{R^2}f w_0 - \rho_0 h\frac{\omega^2}{c}\psi w_0 = 0 \tag{3.1}$$

We substitute the expressions for f, ψ, w_0, given by (2.3) and (2.4), into Eq. (3.1), equate the sum of the free terms to zero, and then equate the sum of the coefficients of $\cos\lambda_n s$ and $\sin\lambda_n s$ to zero. From the condition for the free terms to be equal to zero we obtain

$$c_0 = d_1 = 0 \tag{3.2}$$

From the condition for the coefficients of $\cos\lambda_n s$ and $\sin\lambda_n s$ to be equal to zero, taking (3.2) into account, we obtain the following infinite system of sequential equations:

for $n = 1$

$$\begin{gathered}\left(\xi_1 + \frac{1-\nu^2}{\varsigma_1^2} - \eta_1\right)c_1 + \left(\frac{1-\nu^2}{2\varsigma_1^2}\varepsilon_1 - \frac{\varepsilon_2}{2}\eta_1\right)d_2 = 0,\\ \left(\frac{1-\nu^2}{\varsigma_1^2}\varepsilon_1 - \varepsilon_2\eta_1\right)c_2 = 0\end{gathered} \tag{3.3}$$

for $n \ge 2$

$$\begin{gathered}\xi_n\left(c_n + \frac{\varepsilon_1}{2}\frac{\lambda_{n+1}^2}{\lambda_n^2}d_{n+1} - \frac{\varepsilon_1}{2}\frac{\lambda_{n-1}^2}{\lambda_n^2}d_{n-1}\right) + \frac{1-\nu^2}{\varsigma_n^2}\left(c_n - \frac{\varepsilon_1}{2}d_{n-1} + \frac{\varepsilon_1}{2}d_{n+1}\right) -\\ -\eta_n\left(c_n - \frac{\varepsilon_2}{2}d_{n-1} + \frac{\varepsilon_2}{2}d_{n+1}\right) = 0 \quad (c,d) \to (d,-c)\end{gathered} \tag{3.4}$$

In the first approximation, we obtain from (3.3) a single equation in c_1, whence we also find

$$\eta_1 = \xi_1 + \frac{1-\nu^2}{\varsigma_1^2} \tag{3.5}$$

which is identical with the first frequency of natural vibrations of the mainly transverse vibrations of a uniform cylindrical shell.

The truncated system of equations in the second approximation has the form

$$\left(\xi_1 + \frac{1-\nu^2}{\varsigma_1^2} - \eta_1\right)c_1 + \left(\frac{1-\nu^2}{2\varsigma_1^2}\varepsilon_1 - \frac{\varepsilon_2}{2}\eta_1\right)d_2 = 0$$

$$\left(\xi_2\frac{\varepsilon_1^2}{8} + \frac{1-\nu^2}{2\varsigma_2^2}\varepsilon_1 - \frac{\varepsilon_2}{2}\eta_2\right)c_1 + \left(\xi_2 + \frac{1-\nu^2}{\varsigma_2^2} - \eta_2\right)d_2 = 0 \tag{3.6}$$

$$\left(\frac{1-\nu^2}{\varsigma_1^2}\varepsilon_1 - \varepsilon_2\eta_1\right)c_2 = 0, \quad \left(\xi_2 + \frac{1-\nu^2}{\varsigma_2^2} - \eta_2\right)c_2 = 0 \tag{3.7}$$

By equating the determinant of system (3.6) to zero, we determine the first two vibration frequencies, taking the non-uniformity into account. It follows from Eq. (3.7) that $c_2 = 0$.

If we neglect the quantities ξ_1, ξ_2 in system (3.6), i.e. we consider a momentless shell, then, to determine the first two frequencies, we obtain a quadratic equation, the solution of which has the form

$$\eta_{11} = \frac{1-\nu^2}{\varsigma_1^2}\left(1 - \frac{\varepsilon_1}{2}\right)\left(1 - \frac{\varepsilon_2}{2}\right)^{-1}, \quad \eta_{12} = \frac{1-\nu^2}{\varsigma_1^2}\left(1 + \frac{\varepsilon_1}{2}\right)\left(1 + \frac{\varepsilon_2}{2}\right)^{-1} \tag{3.8}$$

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